Recent results using coverings

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(Supervised research with students from the 2012 Cornell Summer Mathematics Institute)

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Collaborators

Student research groups:

- Lane Bloome, Justin Ferguson, Marcella Noorman
- Kelly Dougan, Mahadi Osman, Jason Tata
- Kelsey Houston-Edwards, Erin Linebarger, Michael Lugo
- Laura Lyman, Tim Morris, Bridget Toomey

Graduate research assistant:

- Elizabeth Wesson

Covering

Def.: Covering system

A *covering system* or covering, for short, is a finite system of congruences

$$n \equiv a_i \pmod{m_i}, \quad 1 \leq i \leq t, \quad 1 < t \in \mathbb{N}$$

such that every integer satisfies at least one of the congruences.

Example of a covering

Erdős (1950):

- $n \equiv 0 \pmod{2}$
- $n \equiv 0 \pmod{3}$
- $n \equiv 1 \pmod{4}$
- $n \equiv 3 \pmod{8}$
- $n \equiv 7 \pmod{12}$
- $n \equiv 23 \pmod{24}$

Basic idea:

- $m_i := \mathrm{ord}_{p_i}(b)$
- p_i to be distinct
- use the Chinese remainder theorem.

Early covering results

- Erdős (1950): $k - 2^n$, minimum modulus problem, odd covering problem
- Riesel (1956): $k2^n - 1$
- Sierpiński (1960): $k2^n + 1$
- Krukenberg (1972): minimum modulus problem
- Cohen and Selfridge (1975): Riesel-Sierpiński numbers
- Brillhart, Lehmer, Selfridge, Tuckerman and Wagstaff (1983, 1988): computed tables of primes for different b.

Coverings in the new millennium

- Myerson*, Poon and Simpson (2007, 2009): incongruent restricted disjoint covering systems
- Gibson* (2009): minimum modulus problem
- Nielsen (2009): minimum modulus problem
- Filaseta, Nicol, Selfridge, K.* (2010): composites that remain composite after changing a digit
- Emanuel* (PhD Thesis 2011, 2012): incongruent restricted disjoint covering systems
- Jones; Jones & White (2011): appending digits to generate an infinite sequence of composite numbers

* Denotes an "appearance" at WCNT.

Recent results using coverings

Summer Mathematics Institute 2012 at Cornell University

- Lane Bloome, Justin Ferguson, and Marcella Noorman: *Appending digits to Sierpiński and Riesel numbers*
- Kelly Dougan, Mahadi Osman, and Jason Tata: *Composites in different bases that remain composite after changing digits*
- Kelsey Houston-Edwards, Erin Linebarger, and Michael Lugo: *Minimality questions inspired by Erdős' minimum modulus problem*
- Laura Lyman, Tim Morris, and Bridget Toomey: *Incongruent restricted disjoint covering systems*

Appending digits to Sierpiński and Riesel numbers

Lane Bloome, Justin Ferguson, Marcella Noorman (and Elizabeth Wesson)

- Motivated by Jones (2011), Jones and White (2011) *Appending digits to generate an infinite sequence of composite numbers*.
  - Study sequences $\{k, k1, k11, k111, \dots\}$ where each term is composite.
  - Write the ith term of the sequence, $k(i)$, as $k \cdot 10^{i+1} + \frac{10^i - 1}{9}$.
  - Pick $m_j = \mathrm{ord}_{p_j}(10)$.
  - $\{p_j\}$ is a finite set of primes such that each term in the sequence is divisible by at least one of these primes.

Appending digits to Sierpiński and Riesel numbers

Theorem

There exist infinitely many Riesel-Sierpiński numbers K such that each term in the sequences $\{K, K1, K11, \ldots\}$, $\{K, K3, K33, \ldots\}$, $\{K, K7, K77, \ldots\}$, $\{K, K9, K99, \ldots\}$ is composite.

Theorem

There exists an infinite subsequence of $\{S, S1, S11, \ldots\}$ where S and each term in the subsequence is a Sierpiński number.

Composites in different bases that remain composite after changing digits

Kelly Dougan, Mahadi Osman, and Jason Tata

- Motivated by Filaseta, Nicol, Selfridge, K. (2010)
  - Construct an infinite sequence of composite numbers, coprime with 10, that remain composite if you change any of their digits.
  - This infinite sequence is an infinite subsequence of $\{M, 1M, 11M, 111M, \ldots\}$ where $M = d_{t-1} \ldots d_0$.
  - $M(0) = M$, $M(1) = 1M = \frac{10^{(t+1)+1}-1}{9} + M'$, ..., $M(j) = \underbrace{1\ldots1}_{j-many 1s} M = \frac{10^{(t+j)+1}-1}{9} + M'$, where M' is t-digits long.
  - Not all primes for our $\{p_i\}$ come from coverings.

Composites in different bases that remain composite after changing digits

Theorem

For each base, $b = 2$ through $b = 9$, there exist infinitely many composite numbers coprime to b that remain composite when you change any digit in its base-b expansion.

- Studied what happens when you change two adjacent digits with dd. (Trivial)
- Study what happens if you change two adjecent digits with $d_1 d_2$ where $d_1, d_2 \in \{0, \ldots, d-1\}$. (Not trivial.)
  - Theoretical results for $b = 2$ and $b = 3$.
  - "Close" for $b = 4$ and $b = 5$.
  - "Not close" for $b \geq 6$.
  - Computational example only for $b = 2$.

Kelsey Houston-Edwards, Erin Linebarger, and Michael Lugo

- Motivated by Krukenberg's PhD Thesis (1972)
    - What is the least, greatest modulus in a covering with minimum modulus c?
    - For $c = 2$ the LGM = 12.
    - For $c = 3$ the LGM = 36.
    - For $c = 4$ the LGM = 60.
    - For $c = 5$ the LGM is conjectured to be 108.

(Partial results)

- Studied $c = 5$.
- Next candidate LGM is 105.
- Residues are a subset of $\{5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 24, 28, 30, 32, 35, 36, 40, 42, 45, 48, 56, 60, 63, 70, 72, 80, 84, 90, 96, 105\}$.
- Tried theoretical and computational methods.
- Attempts were inconclusive.
- Note: Trifonov will be giving a talk on Krukenberg's least greatest modulus problem at the Joint Meetings.

Incongruent restricted disjoint covering systems

Laura Lyman, Tim Morris and Bridget Toomey

- Motivated by Myerson, Simpson, Poon (2007, 2009) and Emanuel (2011).
    - Incongruent (distinct moduli)
    - Disjoint (every integer satisfies exactly one congruence)
    - A covering of the integers cannot be incongruent AND disjoint (Mirsky-Newman).
    - Restricted (not all the integers, each congruence gets used at least twice)
    - For example:
      1 mod 6, 2 mod 9, 0 mod 3, 0 mod 4, 0 mod 5.
    - Another way to write it: $\{6, 9, 3, 4, 5, 3, 6, 4, 3, 5, 9\}$.

Incongruent restricted disjoint covering systems

(Results)

- Wrote a computer program that builds IRDCS of a given length and with given properties.
- Studied IRDCS that include the sequence of modulie $\{\ldots, 9, 6, 3, \ldots\}$.
- Emanuel conjectured that there exists 9-6-3 IRDCS for all lengths ≥ 18
- Showed 963 IRDCS is not possible for length 18.
- Couldn't find one for length 24; confirmed computationally up to length 500.

Conjecture

There exists a 9-6-3 IRDCS for all lengths ≥ 25.

THANK YOU!