# Reflection theorems for forms + rings

Evan M. O'Dorney, WCNT 2019

Qn (Gauss 1801)

How many integer binary quadratic forms

$$ax^2+bxy+cy^2 \;/\; SL_2\mathbb{Z}$$

of discriminant $D$?

Qn (Eisenstein 1890's)

Same for bin. cubics.

Ans. (Ohno-Nakagawa)

Let

$$h_3(D) = \sum_{\substack{f \text{ bin cubic,} \\ \operatorname{Disc} f = D \,/\, SL_2\mathbb{Z}}} \frac{1}{|\operatorname{Stab}_{SL_2\mathbb{Z}} f|}$$

1 or 3

Let

$$h_3'(D) = \sum_{\substack{f = ax^3+bx^2y+cxy^2+dy^3 \\ \text{s.t. } 3 \mid b, c}} \text{''}$$

Thm (O-N)

$$h_3'(-27D) = \begin{cases} h_3(D) & D<0 \\ 3h_3(D) & D>0. \end{cases}$$

Cor. Scholz refl (1930)

$$Cl(\mathbb{Q}(\sqrt{D}))[3] \sim Cl(\mathbb{Q}(\sqrt{-3D}))[3]$$

---

New approach: 2 ideas of Tate

- Fourier analysis on adelic ...
- Tate pairing/Poitou-Tate duality

$$H^1(K, M) \qquad M = (\mathbb{Z}/3\mathbb{Z})(\chi_D)$$

$$M' \simeq (\mathbb{Z}/3\mathbb{Z})(\chi_{-27D})$$

$$H^1(\mathbb{A}_K, M) \times H^1(\mathbb{A}_K, M') \to \mu$$

$$\bigoplus{}' H^1(K_v, M)$$

---

More reflections for:

- $/\mathcal{O}_K$, fn. flds

$S_3 \cong \mathbb{Z}/3\mathbb{Z} \rtimes \mathrm{Aut}(\mathbb{Z}/3\mathbb{Z})$

$S_4$

$D_5 \leftarrow\cdots\rightarrow F_{20}$